

Decoherence in non integrable systems.

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Self-induced decoherence formalism and the corresponding classical limit are extended from quantum integrable systems to non-integrable ones.

I. INTRODUCTION.

Decoherence was initially considered to be produced by *destructive interference* [1]. Later the strategy changed and decoherence was explained as caused by the interaction with an environment [2], but this approach is not conclusive because:

- i.- The environment cannot always be defined, e. g. in closed system like the universe.
- ii.- There is not a clear definition of the "cut" between the proper system and its environment.
- iii.- The definition of the *pointer basis* is not simple.

So we need a new and complete theory: *The self-induced approach* [3], based in a new version of destructive interference, which will be explained in this talk in its version for non-integrable systems. The essential idea is that this interference is embodied in Riemann-Lebesgue theorem where it is proved that if $f(\nu) \in \mathbb{L}_1$ then

$$\lim_{t \rightarrow \infty} \int_{-a}^a f(\nu) e^{-i \frac{\nu t}{\hbar}} dt = 0$$

If we use this formula in the case when $\nu = \omega - \omega'$, where ω, ω' are the indices of the density operator $\hat{\rho}$, in such a way that $\nu = 0$ corresponds to the diagonal, we obtain a *catastrophe*, since all *diagonal* and *not diagonal* terms would disappear. But, if $f(\nu) = A\delta(\nu) + f_1(\nu)$, where now $f_1(\nu) \in \mathbb{L}_1$, we have

$$\lim_{t \rightarrow \infty} \int_{-a}^a f(\nu) e^{-i \frac{\nu t}{\hbar}} dt = A$$

and the diagonal terms $\nu = 0$ remain while the off-diagonal ones vanish. This is the trick we will use below.

II. WEYL-WIGNER-MOYAL MAPPING.

Let $\mathcal{M} = \mathcal{M}_{2(N+1)} \equiv \mathbb{R}^{2(N+1)}$ be the phase space. The functions over \mathcal{M} will be called $f(\phi)$, where ϕ symbolizes the coordinates of \mathcal{M}

$$\phi^a = (q^1, \dots, q^{N+1}, p_q^1, \dots, p_q^{N+1})$$

Then the Wigner transform reads

$$\text{symp} \hat{f} \doteq f(\phi) = \int \langle q + \Delta | \hat{f} | q - \Delta \rangle e^{i \frac{2\Delta}{\hbar}} d^{N+1} \Delta$$

where $\hat{f} \in \hat{\mathcal{A}}$ and $f(\phi) \in \mathcal{A}$ where $\hat{\mathcal{A}}$ is the quantum algebra and the classical one is \mathcal{A} . We can also introduce the star product

$$\text{symp}(\hat{f}\hat{g}) = \text{symp} \hat{f} * \text{symp} \hat{g} = (f * g)(\phi), \quad (f * g)(\phi) = f(\phi) \exp \left(-\frac{i\hbar}{2} \overleftarrow{\partial}_a \omega^{ab} \overrightarrow{\partial}_b \right) g(\phi)$$

and the *Moyal bracket*, which is the symbol corresponding to the commutator

$$\{f, g\}_{mb} = \frac{1}{i\hbar} (f * g - g * f) = \text{symp} \left(\frac{1}{i\hbar} [f, g] \right)$$

so we have

$$(f * g)(\phi) = f(\phi)g(\phi) + 0(\hbar), \quad \{f, g\}_{mb} = \{f, g\}_{pb} + 0(\hbar^2) \quad (1)$$

To obtain the inverse symb^{-1} we will use the *symmetrical* or *Weyl* ordering prescription, namely

$$\text{symb}^{-1}[q^i(\phi)p^j(\phi)] = \frac{1}{2}(\hat{q}^i\hat{p}^j + \hat{p}^j\hat{q}^i)$$

Then we have an isomorphism between the quantum algebra $\hat{\mathcal{A}}$ and the classical one \mathcal{A}

$$\text{symb}^{-1} : \mathcal{A} \rightarrow \hat{\mathcal{A}}, \quad \text{symb} : \hat{\mathcal{A}} \rightarrow \mathcal{A}$$

The mapping so defined is the *Weyl-Wigner-Moyal symbol*.

For the state we have

$$\rho(\phi) = \text{symb}\hat{\rho} = (2\pi\hbar)^{-N-1} \text{symb}_{(\text{for operators})}\hat{\rho}$$

and it turns out that

$$(\hat{\rho}|\hat{O}) = (\text{symb}\hat{\rho}|\text{symb}\hat{O}) = \int d\phi^{2(N+1)} \rho(\phi) O(\phi) \quad (2)$$

Namely the definition $\hat{\rho} \in \hat{\mathcal{A}}$, as a functional on $\hat{\mathcal{A}}$, is equal to the definition $\text{symb}\hat{\rho} \in \mathcal{A}'$, as a functional on \mathcal{A} .

III. DECOHERENCE IN NON INTEGRABLE SYSTEMS.

A. Local CSCO.

a.- When our quantum system is endowed with a CSCO of $N+1$ observables, containing \hat{H} , the underlying classical system is *integrable*. In fact, let $N+1$ -CSCO be $\{\hat{H}, \hat{O}_1, \dots, \hat{O}_N\}$ the Moyal brackets of these quantities are

$$\{O_I(\phi), O_J(\phi)\}_{mb} = \text{symb}\left(\frac{1}{i\hbar}[\hat{O}_I, \hat{O}_J]\right) = 0$$

where $I, J, \dots = 0, 1, \dots, N$ and $\hat{H} = \hat{O}_0$. Then when $\hbar \rightarrow 0$ from eq. (1) we know that

$$\{O_I(\phi), O_J(\phi)\}_{pb} = 0 \quad (3)$$

then as $H(\phi) = O_0(\phi)$ the set $\{O_I(\phi)\}$ is a complete set of $N+1$ constants of the motion in involution, globally defined over all \mathcal{M} , and therefore the system is integrable. q. e. d.

b.- If this is not the case $N+1$ constants of the motion in involution $\{H, O_1, \dots, O_N\}$ *always exist locally*, as can be shown integrating the system of equations (3). Then, if $\phi_i \in \mathcal{M}$ there is *maximal domain of integration* \mathcal{D}_{ϕ_i} around $\phi_i \in \mathcal{M}$ where these constants are defined. In this case the system is *non-integrable*. Moreover we can repeat the procedure with the system

$$\{O_I(\phi), O_J(\phi)\}_{mb} = 0 \quad (4)$$

Then we can extend the definition of the constant $\{H, O_1, \dots, O_N\}$, defined in each \mathcal{D}_{ϕ_i} , outside \mathcal{D}_{ϕ_i} as null functions. Their Weyl transforms $\{\hat{H}, \hat{O}_1, \dots, \hat{O}_N\}$ can be considered as a local $N+1$ -CSCOs related each one with a domain \mathcal{D}_{ϕ_i} that we will call $\{\hat{H}, \hat{O}_{1\phi_i}, \dots, \hat{O}_{N\phi_i}\}$ (we consider that \hat{H} is always globally defined).

c.- We also can define an *ad hoc positive partition of the identity*

$$1 = I(\phi) = \sum_i I_{\phi_i}(\phi)$$

where $I_{\phi_i}(\phi)$ is the *characteristic* function or *index* function, i. e.:

$$I_{\phi_i}(\phi) = \begin{cases} 1 & \text{if } \phi \in \mathcal{D}_{\phi_i} \\ 0 & \text{if } \phi \notin \mathcal{D}_{\phi_i} \end{cases}$$

where the domains $D_{\phi_i} \subset \mathcal{D}_{\phi_i}$ $D_{\phi_i} \cap D_{\phi_j} = \emptyset$. Then $\sum_i I_{\phi_i}(\phi) = 1$. Then we can define $A_{\phi_i}(\phi) = A(\phi)I_{\phi_i}(\phi)$ and

$$A(\phi) = \sum_i A_{\phi_i}(\phi)$$

and using $symb^{-1}$

$$\hat{A} = \sum_i \hat{A}_{\phi_i}$$

We can further decompose

$$\hat{A}_{\phi_i} = \sum_j A_{j\phi_i} |j\rangle_{\phi_i} \langle j|_{\phi_i} \quad (5)$$

where the $|j\rangle_{\phi_i}$ are the corresponding eigenvectors of the local $N+1$ -CSCO of $D_{\phi_i} \subset \mathcal{D}_{\phi_i}$ where a local $N+1$ -CSCO is defined.. So

$$\hat{A} = \sum_{ij} A_{j\phi_i} |j\rangle_{\phi_i} \langle j|_{\phi_i}$$

all over \mathcal{M} . It can be proved that for $i \neq k$ it is

$$\langle j|_{\phi_i} |j\rangle_{\phi_k} = 0$$

so the last decomposition is orthonormal, thus decomposition (5) generalizes the usual eigen-decomposition of integrable system to the non-integrable case. We will use this decomposition below.

B. Decoherence in the energy.

a.- Let us define in each D_{ϕ_i} a local $N+1$ -CSCO $\{\hat{H}, \widehat{O_{\phi_i}}\}$ (as we have said we consider that \hat{H} is always globally defined) as

$$\hat{H} = \int_0^\infty \omega \sum_{im} |\omega, m\rangle_{\phi_i} \langle \omega, m|_{\phi_i} d\omega, \quad \widehat{O_{\phi_i}} = \int_0^\infty \sum_m O_{m_{I\phi_i}} |\omega, m\rangle_{\phi_i} \langle \omega, m|_{\phi_i} d\omega$$

where we have used decomposition (5). The energy spectrum is $0 \leq \omega < \infty$ and $m_{I\phi_i} = \{m_{1\phi_i}, \dots, m_{N\phi_i}\}, m_{I\phi_i} \in \mathbb{N}$. Therefore

$$\hat{H}|\omega, m\rangle_{\phi_i} = \omega|\omega, m\rangle_{\phi_i}, \quad \widehat{O_{\phi_i}}|\omega, m\rangle_{\phi_i} = O_{m_{I\phi_i}}|\omega, m\rangle_{\phi_i}.$$

where, from the orthonormality of the eigenvector and eq. (5), we have

$$\langle \omega, m|_{\phi_i} |\omega', m'\rangle_{\phi_j} = \delta(\omega - \omega') \delta_{mm'} \delta_{ij}$$

b.- A generic observable, in the orthonormal basis just defined, reads:

$$\hat{O} = \sum_{imm'} \int_0^\infty \int_0^\infty d\omega d\omega' \tilde{O}(\omega, \omega')_{\phi_i mm'} |\omega, m\rangle_{\phi_i} \langle \omega', m'|_{\phi_i}$$

where $\tilde{O}(\omega, \omega')_{\phi_i mm'}$ is a generic *kernel* or *distribution* in ω, ω' . As explained in the introduction, the simplest choice to solve our problem is the van Hove choice [4].

$$\tilde{O}(\omega, \omega')_{\phi_i mm'} = O(\omega)_{\phi_i mm'} \delta(\omega - \omega') + O(\omega, \omega')_{\phi_i mm'} \quad (6)$$

where we have a *singular* and a *regular* term, so called because the first one contains a Dirac delta and in the second one the $O(\omega, \omega')_{\phi_i mm'}$ are ordinary functions of the real variables ω and ω' . As we will see these two parts appear in every formulae below. So our operators belong to an algebra $\hat{\mathcal{A}}$ and they read

$$\hat{O} = \sum_{imm'} \int_0^\infty d\omega O(\omega)_{\phi_i mm'} |\omega, m\rangle_{\phi_i} \langle \omega, m'|_{\phi_i} + \sum_{imm'} \int_0^\infty \int_0^\infty d\omega d\omega' O(\omega, \omega')_{\phi_i mm'} |\omega, m\rangle_{\phi_i} \langle \omega', m'|_{\phi_i}$$

The *observables* are the self adjoint $O^\dagger = O$ operators. These observables belong to a space $\hat{\mathcal{O}} \subset \hat{\mathcal{A}}$. This space has the *basis* $\{|\omega, m, m'\rangle_{\phi_i}, |\omega, \omega', m, m'\rangle_{\phi_i}\}$ defined as:

$$|\omega, m, m'\rangle_{\phi_i} \doteq |\omega, m\rangle_{\phi_i} \langle \omega, m'|_{\phi_i}, \quad |\omega, \omega', m, m'\rangle_{\phi_i} \doteq |\omega, m\rangle_{\phi_i} \langle \omega', m'|_{\phi_i}$$

c.- Let us define the quantum states $\hat{\rho} \in \hat{\mathcal{S}} \subset \hat{\mathcal{O}}'$, where $\hat{\mathcal{S}}$ is a convex set. The basis of $\hat{\mathcal{O}}'$ is $\{(\omega, mm')_{\phi_i}, (\omega\omega', mm')_{\phi_i}\}$ and its vectors are defined as functionals by the equations:

$$(\omega, m, m'|_{\phi_i} | \eta, n, n')_{\phi_j} = \delta(\omega - \eta) \delta_{mn} \delta_{m'n'} \delta_{ij}, \quad (\omega, \omega', m, m'|_{\phi_i} | \eta, \eta', n, n')_{\phi_j} = \delta(\omega - \eta) \delta(\omega' - \eta') \delta_{mn} \delta_{m'n'} \delta_{ij}.$$

and all others $(\cdot|\cdot)$ are zero. Then, a generic quantum state reads:

$$\hat{\rho} = \sum_{imm'} \int_0^\infty d\omega \overline{\rho(\omega)}_{\phi_i mm'} (\omega, mm')_{\phi_i} + \sum_{imm'} \int_0^\infty d\omega \int_0^\infty d\omega' \overline{\rho(\omega, \omega')}_{\phi_i mm'} (\omega\omega', mm')_{\phi_i}$$

We require that:

$$\overline{\rho(\omega, \omega')}_{\phi_i mm'} = \rho(\omega', \omega)_{\phi_i m'm}, \quad \rho(\omega, \omega)_{\phi_i mm} \geq 0, \quad (\hat{\rho}|\hat{I}) = \sum_{im} \int_0^\infty d\omega \rho(\omega)_{\phi_i} = 1, \quad (7)$$

where $\hat{I} = \int_0^\infty d\omega \sum_{im} |\omega, m\rangle_{\phi_i} \langle \omega, m|_{\phi_i}$ is the identity operator. Then, in fact, $\hat{\rho} \in \hat{\mathcal{S}}$, where $\hat{\mathcal{S}}$ is a convex set, and we have

$$\langle \hat{O} \rangle_{\hat{\rho}(t)} = (\hat{\rho}(t)|\hat{O}) = \sum_{imm'} \int_0^\infty d\omega \overline{\rho(\omega)}_{\phi_i mm'} O(\omega)_{\phi_i mm'} + \sum_{imm'} \int_0^\infty d\omega \int_0^\infty d\omega' \overline{\rho(\omega, \omega')}_{\phi_i mm'} e^{i(\omega - \omega')t/\hbar} O(\omega, \omega')_{\phi_i mm'} \quad (8)$$

If we now take the limit $t \rightarrow \infty$ and use the Riemann-Lebesgue theorem, being $O(\omega, \omega')$ and $\overline{\rho(\omega, \omega')}_{\phi_i mm'}$ regular (namely $\overline{\rho(\omega, \omega')}_{\phi_i mm'} O(\omega, \omega') \in \mathbb{L}_1$ in the variable $\nu = \omega - \omega'$), we arrive to

$$\lim_{t \rightarrow \infty} \langle \hat{O} \rangle_{\hat{\rho}(t)} = (\hat{\rho}_*|\hat{O}) = \sum_{imm'} \int_0^\infty d\omega \overline{\rho(\omega)}_{\phi_i mm'} O(\omega)_{\phi_i mm'}$$

or to the *weak limit*

$$W \lim_{t \rightarrow \infty} \hat{\rho}(t) = \hat{\rho}_* = \sum_{imm'} \int_0^\infty d\omega \overline{\rho(\omega)}_{\phi_i mm'} (\omega, m, m')_{\phi_i}$$

where only the diagonal-singular terms remain showing that the *system has decohered* in the energy.

Remarks

i.- It looks like that decoherence takes place without a coarse-graining, or an environment. It is not so, the van Hove choice (6) and the mean value (8) are a restriction of the information as effective as the coarse-graining is to produce decoherence.

ii.- Theoretically decoherence takes place at $t \rightarrow \infty$. Nevertheless, for atomic interactions, the *characteristic decoherence time* is $t_D = 10^{-15} \text{seg}$ [5]. For macroscopic systems this time is even smaller (e.g. 10^{-38}seg). Models with two characteristic times (decoherence and relaxation) can also be considered [6].

C. Decoherence in the other variables.

By a change of basis we can diagonalize the $\overline{\rho(\omega)}_{\phi_i mm'}$ in m and m' :

$$\rho(\omega)_{\phi_i mm'} \rightarrow \rho(\omega)_{\phi_i pp'} = \rho_{\phi_i}(\omega)_p \delta_{pp'}.$$

in a new basis orthonormal $\{|\omega, p\rangle_{\phi_i}\}$. Therefore $\rho_{\phi_i}(\omega)_p \delta_{pp'}$ is now diagonal in all its coordinates in a *final local pointer basis in each D_{ϕ_i}* , which, in the case of the observables is $\{|\omega, p, p'\rangle_{\phi_i}, |\omega, \omega', p, p'\rangle_{\phi_i}\}$ (i. e. essentially $\{|\omega', p'\rangle_{\phi_i}\}$), so in this pointer basis we have obtained a *boolean quantum mechanic with no interference terms* and we have the weak limit:

$$W \lim_{t \rightarrow \infty} \hat{\rho}(t) = \hat{\rho}_* = \sum_{ip} \int_0^\infty d\omega \overline{\rho_{\phi_i}(\omega)}_p (\omega, p, p|_{\phi_i}$$

or in the case of \hat{P} with continuous spectra:

$$W \lim_{t \rightarrow \infty} \hat{\rho}(t) = \hat{\rho}_* = \sum_i \int_0^\infty d\omega \int_{p \in D_{\phi_i}} dp^N \overline{\rho(\omega)_{\phi_i}} (\omega, p, p|_{\phi_i} \quad (9)$$

the only case that we will consider below

IV. THE CLASSICAL STATISTICAL LIMIT.

a.- Let us now take into account the Wigner transforms. *There is no problem for regular operators* which are considered in the standard theory. Moreover these operators are irrelevant since they disappear after decoherence.

b.- So we must only consider the singular ones as

$$\hat{O}_S = \sum_i \int_{p \in D_{\phi_i}} dp^N \int_0^\infty O_{\phi_i}(\omega, p) |\omega, p\rangle_{\phi_i} \langle \omega, p|_{\phi_i} d\omega$$

where now the \hat{P} have continuous spectra. So

$$\hat{O}_S = \sum_i O_{\phi_i}(\hat{H}, \widehat{P_{\phi_i}}) = \sum_i \hat{O}_{S\phi_i}$$

But $\hat{H}, \widehat{P_{\phi_i}}$ commute thus

$$\text{symp} \hat{O}_S = O_S(\phi) = \sum_i O_{\phi_i}(H(\phi), P_{\phi_i}(\phi)) + 0(\hbar^2)$$

and if $O_{\phi_i}(\omega, p) = \delta(\omega - \omega') \delta(p - p')$ we have

$$\text{symp} |\omega', p'\rangle_{\phi_i} \langle \omega', p'|_{\phi_i} = \delta(H(\phi) - \omega') (P_{\phi_i}(\phi) - p)$$

(*really up to $0(\hbar^2)$*), but for the sake of simplicity we will eliminate these symbols from now on).

Let us now consider the singular dual, the $\text{symp} \hat{\rho}_S$ as the functional on \mathcal{M} that must satisfy eq. (2) that now reads

$$(\text{symp} \hat{\rho}_S | \text{symp} \hat{O}_S) = (\hat{\rho}_S | \hat{O}_S)$$

Then we define a density function $\rho_S(\phi) = \text{symp} \hat{\rho}_S = \sum_i \rho_{\phi_i S}(\phi)$ such that

$$\sum_i \int d\phi^{2(N+1)} \rho_{\phi_i S}(\phi) O_{\phi_i S}(\phi) = \sum_i \int_{p \in D_{\phi_i}} \int_0^\infty \rho_{\phi_i}(\omega, p) O_{\phi_i}(\omega, p) d\omega dp^N \quad (10)$$

$\widehat{\rho}_S$ is constant of the motion, so $\rho_{\phi_i}(\phi) = f(H(\phi), P_{\phi_i}(\phi))$. Then we *locally define at D_{ϕ_i}* the local action-angle variables $(\theta^0, \theta^1, \dots, \theta^N, J_{\phi_i}^0, J_{\phi_i}^1, \dots, J_{\phi_i}^N)$, where $J_{\phi_i}^0, J_{\phi_i}^1, \dots, J_{\phi_i}^N$ would just be $H, P_{\phi_i 1}, \dots, P_{\phi_i N}$ and we make the *canonical transformation* $\phi^a \rightarrow \theta_{\phi_i}^0, \theta_{\phi_i}^1, \dots, \theta_{\phi_i}^N, H, P_{\phi_i 1}, \dots, P_{\phi_i N}$ so that

$$d\phi^{2(N+1)} = dq^{(N+1)} dp^{(N+1)} = d\theta_{\phi_i}^{(N+1)} dH dP_{\phi_i}^N$$

Now we will integrate of the functions $f(H, P_{\phi_i}) = f(H, P_{\phi_i}, \dots, P_{\phi_i})$ using the new variables.

$$\int_{D_{\phi_i}} d\phi^{2(N+1)} f(H, P_{\phi_i}) = \int_{D_{\phi_i}} d\theta_{\phi_i}^{(N+1)} dH dP_{\phi_i}^N f(H, P_{\phi_i}) = \int_{D_{\phi_i}} dH dP_{\phi_i}^N C_{\phi_i}(H, P_{\phi_i}) f(H, P_{\phi_i})$$

where we have integrated the angular variables $\theta_{\phi_i}^0, \theta_{\phi_i}^1, \dots, \theta_{\phi_i}^N$, obtaining the *configuration volume* $C_{\phi_i}(H, P_{\phi_i})$ of the portion of the hypersurface defined by $(H = \text{const.}, P_{\phi_i} = \text{const.})$ and contained in D_{ϕ_i} . So eq. (10) reads

$$\sum_i \int_{p \in D_{\phi_i}} \int_0^\infty \rho_{\phi_i}(\omega, p) O_{\phi_i}(\omega, p) d\omega dp^N = \sum_i \int dH dP_{\phi_i}^N C_{\phi_i}(H, P_{\phi_i}) \rho_{\phi_i S}(H, P_{\phi_i}) O_{\phi_i S}(H, P_{\phi_i})$$

for any $O_{\phi_i}(\omega, p)$ so $\rho_{S\phi_i}(H, P) = \frac{1}{C_{\phi_i}} \rho_{\phi_i}(H, P)$ for $\phi \in \mathcal{D}_{\phi_i}$ and

$$\rho_S(\phi) = \rho_*(\phi) = \sum_i \frac{\rho_{\phi_i}(H(\phi), P_{\phi_i}(\phi))}{C_{\phi_i}(H, P_{\phi_i})}$$

Putting $\rho_{\phi_i}(\omega, p) = \delta(\omega - \omega') \delta^N(p - p')$ for some i and all other $\rho_{\phi_j}(\omega, p) = 0$ for $j \neq i$, we have

$$\text{symb}(\omega', p', (\phi)|_{\phi_i}) = \frac{\delta(H(\phi) - \omega') \delta^N(P(\phi) - p')}{C_{\phi_i}(H, P_{\phi_i})}$$

c.- Moreover the *symb* of eq.(9) reads

$$\rho_S(\phi) = \rho_*(\phi) = \sum_i \int_{p \in D_{\phi_i}} dp \int_0^\infty d\omega \rho_{\phi_i}(\omega, p) \frac{\delta(H(\phi) - \omega) \delta^N(P(\phi) - p_{\phi_i})}{C_{\phi_i}(H, P_{\phi_i})} \quad (11)$$

So we have obtained a decomposition of $\rho_*(\phi) = \rho_S(\phi)$ in classical hypersurfaces $(H = \omega, P_{\phi_i}(\phi) = p_{\phi_i})$, containing *chaotic trajectories* (since the system is not integrable), summed with different weight coefficients $\rho_{\phi_i}(\omega, p)/C_{\phi_i}(H, P_{\phi_i})$.

d.- Finally only after decoherence the positive definite diagonal-singular part remains and from eqs. (72) and (11) we see that

$$\rho_{\phi_i}(\omega, p) \geq 0 \Rightarrow \rho_*(\phi) \geq 0$$

so the *classical statistical limit* is obtained.

V. THE CLASSICAL LIMIT.

The classical limit can be decomposed in the following processes

$$\text{Quantum Mechanics} - (\text{decoherence}) \rightarrow \text{Boolean Quantum Mechanics} - (\text{symb and } \hbar \rightarrow 0) \rightarrow$$

$$\text{Classical Statistical Mechanics} - (\text{choice of a trajectory}) \rightarrow \text{Classical Mechanics}$$

where the first two have been explained. It only remains the last one: For $\tau(\phi) = \theta_{\phi_i}^0(\phi)$ and at any fixed t we have

$$\sum_i \int_{D_{\phi_i}} \delta(\tau(\phi) - \tau_0 - \omega t) \delta(\theta_{\phi_i}(\phi) - \theta_{\phi_i 0} - p_{\phi_i} t) d\tau_0 d\theta_{\phi_i 0} = 1$$

then we can include this 1 in decomposition (11) and we obtain

$$\rho_*(\phi) = \sum_i \int \frac{\rho_{\phi_i}(\omega, p_{\phi_i})}{C(\omega, p_{\phi_i})} \delta(H(\phi) - \omega) \delta(P_{\phi_i} - p_{\phi_i}) \delta(\tau(\phi) - \tau_0 - \omega t) \delta(\theta_{\phi_i}(\phi) - \theta_{\phi_i 0} - p_{\phi_i} t) d\omega d^N p_{\phi_i} d\tau_0 d\theta_{\phi_i 0}$$

namely a sum of *classical chaotic trajectories* satisfying:

$$H(\phi) = \omega, \quad \tau(\phi) = \tau_0 + \omega t, \quad P_{\phi_i} = p_{\phi_i}, \quad \theta_{\phi_i}(\phi) = \theta_{\phi_i 0} + p_{\phi_i} t$$

weighted by $\frac{\rho_{\phi_i}(\omega, p_{\phi_i})}{C(\omega, p_{\phi_i})}$, where we can choose any one of them. In this way the classical limit is completed, in fact we have found the classical limit of a quantum system since we have obtained the classical trajectories, so the *correspondence principle* is also obtained as a theorem.

VI. CONCLUSION.

- i.- We have defined the classical limit in the non-integrable case.
 - ii.- Essentially we have presented a *minimal formalism for quantum chaos* [7].
 - iii.- We have deduced the correspondence principle.
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